

## Assignment No. 10

### Due: 13:15 on Tuesday 24 March 2015 via email

There is no word limit/requirement for these exercises. Your responses may be in English, French, Spanish, German, Arabic, or any other language you are comfortable writing in. The grammar, spelling, and prescriptive conventions are not evaluated for the assignment. You do not need to edit, revise a number of times, or attend in any special way to form or language. You should just write in a way that is clear to you. You are welcome to use bullet points. You do not need to write complete sentences or in paragraph form complete with transitions.

Homework should be submitted by 15:15 on the day it is due. There is no late homework accepted. All written assignments must be typed using 12 pt Times New Roman or 11 pt Arial font with 1" margins. All assignments must be send in one of the following formats: .doc, .docx, .txt, .tex, .pdf, .rtf, .odt, .dot. Remember to cite all sources and use APA guidelines. Homework must also include your name, class, date, and assignment.

## 1 Truth Theorems

We are going to be moving on from lecturing on truth tables, formula definitions/theorems, and prose definition-s/theorems for 1. negation, 2. conjunction, 3. disjunction, 4. conditionals, and 5. conditionals. We will no longer be spending class time to cover these topics. You *must* memorize these definitions as these are fundamental to covering other topics in the class and no more class time will be dedicated to motivating these theorems. While you can ascribing the propositional symbols with meaning, you need to just memorize the definitions and take them as axiomatic.

To ensure that we are starting from the same page on Monday, there will be a **quiz** on Monday which will test your ability to reproduce the definitions in the three different formats (e.g. *prose definition*: "A conjunction of two propositional expressions is True iff the two propositional expressions are individually True"; *formula definition*:  $[[\phi \wedge \psi]] = 1$  iff  $[[\phi]] = [[\psi]] = 1$ ; and the *truth table*). If you continue to have questions about these definitions, please email me or make an appointment to come in for office hours.

## 2 Scope

Below there are at least two choices for reading the scope of negation. Remember that you can think about scope as the domain over which the logical connective operates. Draw truth tables for at least two of the scope readings of the following. Additionally, review the lecture slides from lecture 11 and review the section on the main connective. Use this information to complete the homework and for all of the expressions below, state the main connective of the expression. There is no bracketing for the expressions below. When drawing the truth tables, do the bracketing as you see fit which will determine what you consider to be the main connective. Remember that you can think of these expressions as having some meanings, but that is irrelevant to the task at hand. You are just manipulating abstract symbols. Additionally, you are welcome to change the Greek letters to letter in the American English alphabet.

For the complex expression begin by making it simpler by starting with columns for its subparts. For example, begin with a column for  $\phi$  and another one for  $\psi$  (4 rows), then create a column for  $\neg\phi$  (call this column 3 which is the negation of column 1). Now you can create a column for  $(\phi \vee \psi)$  which is calculated based off of the definition of conjunction using the truth values of column 1 and column 2. Then create a fourth column for  $(\neg\phi \wedge \psi)$ , using the truth values in column 3. Finally, make a fifth column for  $(\neg\phi \wedge \psi) \leftrightarrow \psi$  and use the truth values in column 4 in conjunction with  $\psi$  to determine the truth values for the fifth column based on the definition for the biconditional.

1.  $\phi \vee \psi \leftrightarrow \neg\phi \wedge \psi$ .
2.  $\neg\phi \vee \psi \leftrightarrow \phi \rightarrow \psi$
3.  $\phi \wedge \neg\phi \rightarrow \psi$
4.  $\neg\phi \vee \neg\phi$

5.  $\neg\phi \rightarrow \psi \rightarrow \psi \rightarrow \rho \wedge \neg\phi \rightarrow \rho$
6.  $\phi \vee \psi \wedge \neg\phi \rightarrow \psi$

### 3 Truth Tables – Advanced

Using truth tables, show that the following are tautologies (i.e. always true).

1.  $((\neg\phi \vee \psi) \leftrightarrow (\phi \rightarrow \psi))$
2.  $(\neg(\phi \wedge \psi) \leftrightarrow (\neg\phi \vee \neg\psi))$
3.  $(\neg(\phi \vee \psi) \leftrightarrow (\neg\phi \wedge \neg\psi))$
4.  $((\phi \wedge \neg\phi) \rightarrow \psi)$

How do we treat the word **but** in the proposition “The food is cheap but good”? Is this statement equivalent to “The food is cheap and good”? If we can treat “but” as the same as the logical connective  $\wedge$ , the two are equivalent. Verify this using truth tables.

Another question is, do we have an “exclusive or”  $\oplus$  in our semantics? Draw a truth tables for the following and point out the problems.

5.  $p \vee q \wedge \neg(p \wedge q)$
6.  $p \oplus q$
7. “John spoke to Mary or Sue” (using the inclusive and exclusive or)
8. “John spoke to Mary or Sue or Both” (using the inclusive and exclusive or)

In class we talked about how in the propositional logic system we are using there is no way of expressing ordering and the meaning associated with that ordering in conjunctions. This means that  $\phi \wedge \psi$  is equivalent to  $\psi \wedge \phi$ . Come up with *two* English language examples where this is clearly not the case.

### 4 Predicate Logic – Introduction

Read ch. 3 “Getting Inside Sentences” of Gregory (2000). *Semantics*. Complete the following exercises which are adapted from the book.

We will be using a slightly different predicate logic notation which will be introduced in class on Tuesday. For the homework exercises, you may use the notation system in the book, or any other notation system you know of.

You are to do this individually, and not with your group. While the answers to similar questions are in the back of the book, you should do this on your own as using the answers in the book is misleading and I assume that the topic doesn’t need to be covered in class (e.g. the truth table unit).

We have not covered predicate logic in lecture yet but will introduce it on Tuesday. It is important that you do the reading at attempt the homework. Given that the homework is credit/no-credit, it is only important that you *attempt* this homework section. The purpose of this is to ensure that you have read the book and are coming prepared to class. We will begin lecture on Tuesday 24 March 2015 talking about predicate logic and how to write and interpret it. If you have questions after reading the book and completing the homework, email or make an appointment for office hours.

#### 4.1 Exercise 3.8 & 3.13

Translate these sentences into predicate-argument notation.

1. 'Hawai'i is exciting'
2. 'Karen is a genius'
3. 'Mahmoud admires Fairouz'
4. 'Fairouz admires Mahmoud'
5. 'Dana is taller than Janelle'
6. 'Cerberus barks'
7. 'Nishi is a secretary'
8. 'Tina Fey wrote *Bossypants*'
9. 'the North Koreans leaked the movie to the internet'
10. 'Nate thinks that Keith is handsome'
11. 'Graciela told Bertha that Sandra thinks that Maria Elena stole Eduardo from Rosa'

#### 4.2 Exercise 3.9 & 3.14

Translate these predicate-argument formulas into English.

1. crazy(noora)
2. learn(sean, latin)
3. give(christen, ernesto, flowers)
4. father\_of(parley, christen)
5. square\_of(9, 3)
6. play(martina, tennis)
7. bossy(euna)
8. capital\_of(beirut, lebanon)
9. in(tasmania, australia)
10. near(the\_falkland\_islands, argentina)
11. send(dani, sarah, arkansas)

## 5 Entailment

**Updated 21 March 2015:** Read ch. 4 “Meaning Relations (1)” of Gregory (2000). *Semantics*. Complete the following exercises taken from the book. You are to do this individually, and not with your group. While the answers are in the back of the book, you should do this on your own.

In class this past Thursday, in an effort to come up with meaningful examples to review truth tables, the examples below were introduced, however, this turned out to be more confusing as it introduced another level of complexity. In dealing with truth tables, we had been assigning meaning to the abstract propositional symbols (e.g.  $\phi, \psi, \rho, a, b, c$ ) and then given the different permutations of the truth values and resulting truth value of the complex expressions, have been imagining what the state of the world would have to be for those truth values to hold. This gets very confusing because we got into the habit of forcing contexts to make the values work regardless of interpretability. In the exercise below, we are looking instead at the relationship between sentence meanings as is for the world in general.

Entailment is a relationship between propositions that whenever the first proposition is true, the second is true. If there is a situation where the first proposition can be true and the second proposition might not be true, then the first proposition does not entail the second proposition. Therefore we can say that the first proposition entails the second proposition if and only if when the first proposition is true the second proposition is also true. For example, the proposition *all actors are rich* entails the proposition *all famous actors are rich* because we cannot find a case where an actor is not rich, and therefore we could not also find a case where a famous actor is not rich. You can visualize this in terms of a venn diagram or sets. The first proposition (the set of *all actors*) is a superset of the second proposition (the set of *all famous actors*) and therefore there cannot be an entity which is a member of the set of *famous actors* and not a member of the set of *actors*.

For the following statements below, state whether the statement is TRUE or FALSE. Taken from Hurford and Heasley (1983) and modified.

1. *To cook an egg* entails *Having boiled it*.
  - Analogous to the two statements from the lecture *Xochitl cooked an egg* and *Xochitl boiled an egg*.
2. *To boil an egg* entails *Having cooked it*.
  - Analogous to the two statements from the lecture *Ofra boiled an egg* and *Ofra cooked an egg*.
3. *To see a boy* entails *Having seen a person*.
  - Analogous to the two statements from the lecture *I saw a boy* and *I saw a person*.
4. *To steal a car* entails *Having taken it*.
  - Analogous to the two statements from the lecture *Yuliya stole a car* and *Yuliya took a car*.
5. *To eat something* always entail *Having eaten all the cookies in the world*,
  - Analogous to the two statements from the lecture *Sameer ate something* and *Sameer ate all the cookies*.
6. *To kill one's parents* entails *Those same parent's being dead*.
  - Analogous to the two statements from the lecture *Lizzie killed her parents* and *Lizzie's parents died*.
7. *All actors are rich* entails *All famous actors are rich*.
8. *Reza saw two cats in the box* entails *There were two cats in the box*.
9. *Golnesa likes the writers or the actors* entails *Golnesa likes the writers*.
10. *Every student has finished the homework* entails *Every Mexican student has finished the homework*.
11. *Everyone who got at least three A's must skip the next assignment* entails *Everyone who got at least five A's must skip the next assignment*.

12. *The shah of Iran doesn't play tennis* entails *There is a shah of Iran*.
13. *Every Tejana in the room doesn't smoke* entails *There is a Tejana in the room*.